Missing Measurements of Weak-Field Gravity

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For practical and historical reasons, most of what we know about gravity is based on observations made or experiments conducted *beyond the surfaces* of dominant massive bodies. Although the *force* of gravity inside a massive body can sometimes be measured, it remains to demonstrate the *motion* that would be caused by that force through the body's center. Since the idea of doing so has often been discussed as a thought experiment, we here look into the possibility of turning this into a real experiment. Feasibility is established by considering examples of similar experiments whose techniques could be utilized for the present one.

1 Introduction

A recent paper in this journal (M. Michelini [1]) concerned the absence of measurements of Newton's constant, *G*, within a particular range of vacuum pressures. Important as it may be to investigate the physical reasons for this, a gap of equal, if not greater importance concerns the absence of gravity experiments that probe the *motion* of test objects through the centers of larger massive bodies. As is the case for most measurements of *G*, the apparatus for the present experimental idea is also a variation of a torsion balance. Before describing the modifications needed so that a torsion balance can measure through-the-center motion, let's consider the context in which we find this gap in experimentation.

Often found in undergraduate physics texts [2–5] is the following problem, discussed in terms of Newtonian gravity: A test object is dropped into an evacuated hole spanning a diameter of an otherwise uniformly dense spherical mass. One of the reasons this problem is so common is that the answer, the predicted equation of motion of the test object, is yet another instance of *simple harmonic motion*. What is rarely pointed out, however, is that we presently lack direct empirical evidence to verify the theoretical prediction. Confidence in the prediction is primarily based on the success of Newton's theory for phenomena that test the *exterior* solution. Extrapolating Newton's law to the interior is a worthwhile mathematical excercise. But a theoretical extrapolation is of lesser value than an empirical fact.

Essentially the same prediction follows from general relativity [6–9]. In this context too, the impression is sometimes given that the predicted effect is a physical fact. A noteworthy example is found in John A. Wheeler's book, *"A Journey into Gravity and Spacetime"*, in which he refers to the phenomenon as "boomeranging". Wheeler devotes a whole (10-page) chapter to the subject because, as he writes, "Few examples of gravity at work are easier to understand in Newtonian terms than boomeranging. Nor do I know any easier doorway to Einstein's concept of gravity as manifestation of spacetime curvature" [10]. But nowhere in Wheeler's book is there any discussion of *empirical evidence* for "boomeranging". No doubt, Newton, Einstein and Wheeler would all have been delighted to see the simple harmonic motion demonstrated as a laboratory experiment.

2 Feasibility

Since the predicted effect has never been observed at all, our initial goal should simply be to ascertain that the oscillation prediction is a correct approximation. After laying out a basic strategy for doing the experiment, this paper concludes with a few additional remarks concerning motivation.

Apparatus that would have sufficed for our purpose were considered in the 1960s–1970s to measure *G*. Y. T. Chen discusses these through-the-center oscillation devices in his 1988 review paper on *G* measurements [11]. Each example in this group of proposals was intended for space-borne satellite laboratories. The original motivation for these ideas was to devise ways to improve the accuracy of our knowledge of *G* by timing the oscillation period of the simple harmonic motion. Though having some advantages over Earth-based *G* measurements, they also had drawbacks which ultimately prohibited them from ever being carried out.

What distinguishes these proposals from experiments that have actually been carried out in Earth-based laboratories is that the test objects were to be allowed to fall freely back and forth between extremities inside a source mass the whole time. Whereas *G* measurements conducted on Earth typically involve restricting the test mass's movement and measuring the force needed to do so. The most common, and historically original, method for doing this is to use a torsion balance in which a fiber provides a predetermined resistance to rotation. Torsion balances have also been used to test Einstein's Equivalence Principle (e.g. Gundlach et al. [12]). Another distinguishing characteristic of Earth-based *G* measurements and Equivalence Principle tests is that the test masses typically remain outside the larger source masses. Since movement of the test masses is restricted to a small range of motion, these tests can be characterized as static measurements. Torsion balance experiments in which the test mass is inside the source mass have also been performed (for example, Spero

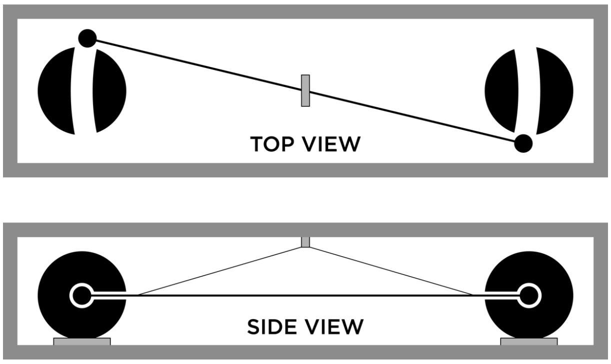

Fig. 1: Schematic of modified Cavendish balance. Since the idea is to demonstrate the simple harmonic motion only as a first approximation, deviation due to the slight arc in the trajectory is inconsequential.

et al. [13] and Hoskins et al. [14]). These latter experiments were tests of the inverse square law.

All three of these types of experiments — G measurements, Equivalence Principle and inverse-square law tests — however, are *static* measurements in the sense that the test masses were not free to move beyond a small distance compared to the size of the source mass. The key innovation in the present proposal is that we want to see an object fall radially as long as it will; we want to eliminate (ideally) or minimize (practically) any obstacle to the radial free-fall trajectory. Space-based experiments would clearly be the optimal way to achieve this. But a reasonably close approximation can be achieved with a modified Cavendish balance in an Earth-based laboratory.

As implied above, the key is to design a suspension system which, instead of providing a restoring force that prevents the test masses from moving very far, allows unrestricted or nearly unrestricted movement. Two available possibilities are fluid suspensions and magnetic suspensions (or a combination of these). In 1976 Faller and Koldewyn succeeded in using a magnetic suspension system to get a G measurement [15, 16]. The experiment's accuracy was not an improvement over that gotten by other methods, but was within 1.5% of the standard value.

As Michelini pointed out in his missing vacuum range discussion, in most G measurements the source masses are located outside an enclosure. Even in the apparatus Cavendish used for his original G measurement at atmospheric pressure, the torsion arm and test masses were isolated from the source masses by a wooden box. In Faller and Koldewyn's experiment the arm was isolated from the source masses by a vacuum chamber. The modified design requires that there be no such isolation, as the arm needs to swing freely through the center of the source masses (see Fig. 1). Given the modest goal of the present proposal, it is reasonable to expect that the technology used by Faller and Koldewyn could be adapted to test the oscillation prediction. Moreover, it seems reasonable to expect that advances in technology since 1976 (e.g. better magnets, better electronics, etc.), would make the experiment quite doable for an institution grade physics laboratory.

3 Motivation: Completeness and Aesthetics

One hardly needs to mention the many successes of Newtonian gravity. By success we mean, of course, that empirical observations match the theoretical predictions. Einsteinian gravity is even more successful. The purpose of many contemporary gravity experiments is to detect physical manifestations of the differences between Newton's and Einstein's theories. In every case Einstein's theory has proven to be more accurate. This is impressive. Given the level of thoroughness and sophistication in gravity experimentation these days, one may be taken aback to realize that Newton's and Einstein's theories *both remain untested* with regard to the problem discussed above. The simple harmonic motion prediction is so common and so obvious that we have come to take it for granted. When discussing the *prediction* for this basic experiment in weak field gravity, it would surely be more satisfactory if we could at the same time cite the *physical evidence*.

The Newtonian explanation for the predicted harmonic motion is that a massive sphere produces a force (or potential) of gravitational attraction. The corresponding general relativistic explanation is that the curvature of spacetime causes the motion. Specifically, the predicted effect is due to the slowing of clock rates toward the center of the sphere. A physical demonstration of the effect would thus indirectly, though convincingly, support general relativity's prediction that the rate of a clock at the body's center is a local minimum — a prediction that has otherwise not yet been confirmed.

In summary, if R represents the surface of a spherical mass, our empirical knowledge of how things move because of the mass within R is essentially confined to the region, $r \gtrsim R$. The region $0 \leq r \lesssim R$ is a rather fundamental and a rather large gap. It is clearly the most ponderable part of the domain. Why not fill this gap?

One of the distinctive features of the kind of experiment proposed above is that its result is, in principle, independent of size. The satellite versions mentioned by Chen were thus referred to as "clock mode" experiments. The determining factor in the oscillation period is the density of the source mass. If the source mass is made of lead (density, $\rho \approx 11,000$ kg/m^3) the oscillation period is about one hour. Would it not be fascinating to observe for an hour, to watch the oscillation take place, knowing that the mass of the larger body is the essential thing making it happen? In my opinion this would be a beautiful sight. Beautiful for completing the domain, $0 \leq r \lesssim R$, and beautiful simply to see what no human being has seen before.

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